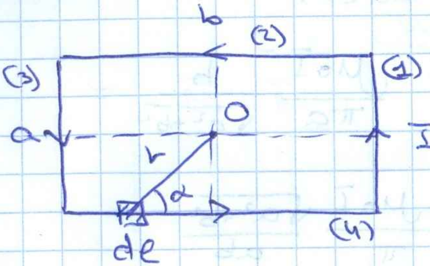
$\boxed{7.1}$ $\boxed{7}$



$$dB = \frac{\mu_0 I}{4\pi}\,\frac{dl \cdot r \cdot \sin\alpha}{r^3} =$$

$$= \left\{ \begin{array}{l} r\sin\alpha = h_1 \\ dl\sin\alpha = r\,d\alpha = \dfrac{h\,d\alpha}{\sin\alpha} \end{array} \right\} =$$

$$= \frac{\mu_0 I}{4\pi}\cdot\frac{\sin^2\alpha}{h^2}\cdot\frac{h\,d\alpha}{\sin\alpha} = \frac{\mu_0 I}{4\pi h}\sin\alpha\,d\alpha$$

$$B_1(O) = B_3(O) = \left\{ tg\alpha_1 = \frac{b}{a};\ h = \frac{b}{2} \right\} =$$

$$= \int_{\alpha_1}^{\pi-\alpha_1} \frac{\mu_0 I}{4\pi h}\sin\alpha\,d\alpha = 2\cdot\frac{\mu_0 I}{4\pi}\cdot\frac{2}{b}\cdot\cos\alpha_1 =$$

$$= \left\{ tg^2\alpha_1 + 1 = \frac{1}{\cos^2\alpha_1} \Rightarrow \cos\alpha_1 = \frac{a}{\sqrt{a^2+b^2}} \right\} =$$

$$= \frac{\mu_0 I}{\pi b}\cdot\frac{a}{\sqrt{a^2+b^2}}$$

$$B_2(O) = B_4(O) = \left\{ tg\alpha_2 = \frac{a}{b};\ h = \frac{a}{2} \right\} =$$

$$= \int_{\alpha_2}^{\pi-\alpha_2} \frac{\mu_0 I}{4\pi h}\sin\alpha\,d\alpha = 2\cdot\frac{\mu_0 I}{4\pi}\cdot\frac{2}{a}\cdot\cos\alpha_2 =$$

$$= \left\{ tg^2\alpha_2 + 1 = \frac{1}{\cos^2\alpha_2} \Rightarrow \cos\alpha_2 = \frac{b}{\sqrt{a^2+b^2}} \right\} =$$

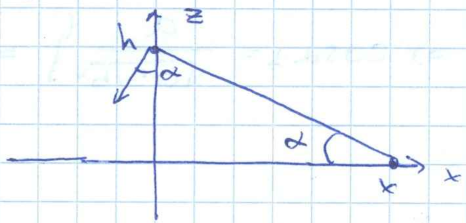
$$= \frac{\mu_0 I}{a}\cdot\frac{b}{\sqrt{a^2+b^2}}$$

$$B(O) = B_1(O) + B_2(O) + B_3(O) + B_4(O) =$$

$$= 2 \cdot \frac{\mu_0 I}{\pi b} \frac{a}{\sqrt{a^2+b^2}} + 2 \cdot \frac{\mu_0 I}{\pi a} \frac{b}{\sqrt{a^2+b^2}} =$$

$$= \frac{2\mu_0 I}{\pi\sqrt{a^2+b^2}} \left(\frac{a}{b} + \frac{b}{a} \right) = \frac{2\mu_0 I}{\pi} \frac{\sqrt{a^2+b^2}}{ab}$$

$\boxed{7.2}$ Проведём на обтекаемой током плоскости линию, перпенд. к направл. тока. Ток, приходящ. на един. длины такой линии, называется линейной плотностью тока, и рассматривается как вектор $\vec{i}$ направленный вдоль тока.



Разобьём плоскость на тонкие нити, тогда

$$dB = \frac{\mu_0 i \, dx}{2\pi\sqrt{x^2+h^2}}$$

$$\cos\alpha = \frac{x}{\sqrt{x^2+h^2}} \qquad \sin\alpha = \frac{h}{\sqrt{x^2+h^2}}$$

$$B_x = \int_{-\infty}^{+\infty} dB \sin\alpha = \frac{\mu_0 i}{2\pi} h \int_{-\infty}^{+\infty} \frac{dx}{x^2+h^2} =$$

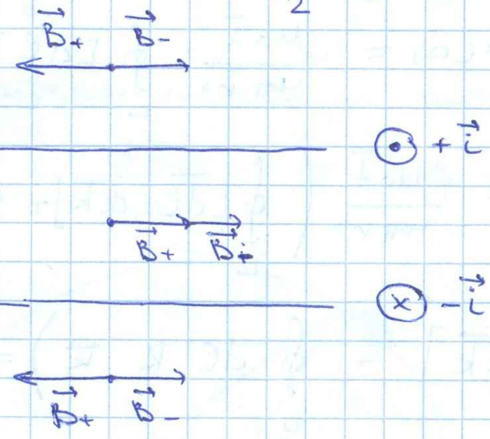
$$= \frac{\mu_0 i}{2\pi} \cdot h \cdot \left[\frac{1}{h} \operatorname{arctg} \frac{x}{h} \right] \Bigg|_{-\infty}^{+\infty} = \frac{\mu_0 i}{2}$$

$$B_y = \int_{-\infty}^{+\infty} dB \cos\alpha = \frac{\mu_0 i}{2\pi} \int_{-\infty}^{+\infty} \frac{x\,dx}{x^2+h^2} = \frac{\mu_0 i}{4\pi} \ln(a^2+x^2) \Big|_{-\infty}^{+\infty}$$

$$= 0$$

7.3 из зад. 7.2 известно:

$$B = \frac{\mu_0 i}{2}; \quad \vec{B} = \frac{\mu_0 [\vec{i}, \vec{n}]}{2}$$



поле вне пластин:

$$\vec{B} = \vec{B}_+ + \vec{B}_- = \frac{\mu_0 [+\vec{i}, \vec{n}]}{2} + \frac{\mu_0 [-\vec{i}, \vec{n}]}{2} = 0$$

поле внутри пластин:

$$\vec{B} = \vec{B}_+ + \vec{B}_- = \frac{\mu_0 [\vec{i}, \vec{n}]}{2} + \frac{\mu_0 [-\vec{i}, -\vec{n}]}{2} =$$

$$= \mu_0 [\vec{i}, \vec{n}].$$

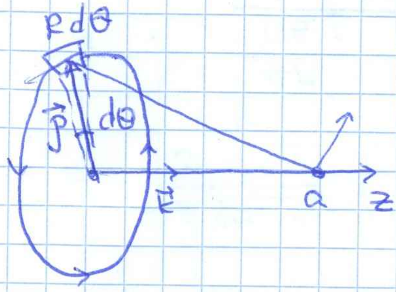
7.7 аналогично задаче 7.2

$$dB = \frac{\mu_0 i \, dx}{2\pi \sqrt{x^2 + h^2}}; \quad \cos\alpha = \frac{x}{\sqrt{x^2+h^2}}; \quad \sin\alpha = \frac{h}{\sqrt{x^2+h^2}}$$

$$B_x = \int_{-l/2}^{l/2} dB \sin\alpha = \frac{\mu_0 i}{2\pi} \operatorname{arctg} \frac{x}{h} \Big|_{-l/2}^{l/2} = \frac{\mu_0 i}{\pi} \operatorname{arctg} \frac{l}{2h}$$

$$B_y = \int_{-l/2}^{l/2} dB \cos\alpha = \frac{\mu_0 i}{4\pi} \ln (h^2 + x^2) \Big|_{-l/2}^{l/2} = 0.$$

$\boxed{7.6}$ 1) Вычислим магнитную индукцию $\vec{B}$ кольца:



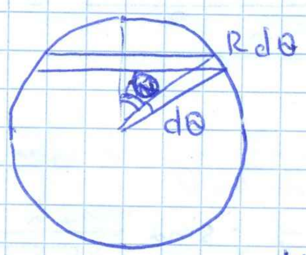
$$\vec{B}(a) = \frac{\mu_0 I}{4\pi r^3} \oint_L [\vec{dl}, \vec{r}] =$$

$$= \frac{\mu_0 I}{4\pi r^3} \left(\oint_L [\vec{dl}, a\vec{k}] + \oint_L [\vec{dl}, \vec{\rho}] \right) =$$

$$= \frac{\mu_0 I}{4\pi r^3} \left(\underbrace{\left[\oint_L \vec{dl}, a\vec{k} \right]}_{=0} + \oint_L dl \cdot R \cdot \vec{k} \right) =$$

$$= \frac{\mu_0 I}{4\pi r^3} \vec{k} \oint_0^{2\pi} R \cdot R d\Theta = \frac{\mu_0 I R^2}{2 r^3} = \frac{\mu_0 I R^2}{2 (R^2 + a^2)^{3/2}}$$

2) Разобьём сферу на кольца:



$$dI = \frac{dq}{T} = \frac{\sigma dS}{T} = \frac{\sigma \cdot 2\pi R \sin\Theta \cdot R d\Theta}{T} =$$

$$= \sigma \omega R^2 \sin\Theta \, d\Theta$$

Можно найти $\vec{B}$ в центре сферы от кольца:

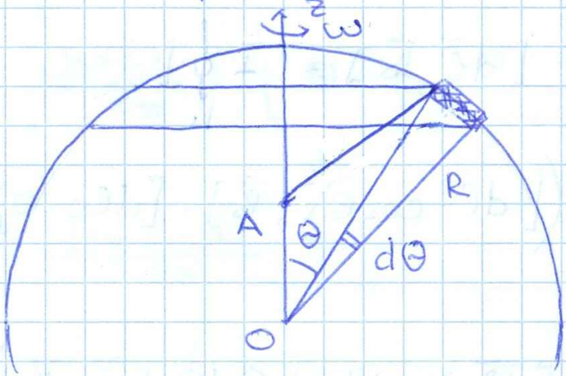
$$dB_0 = \frac{\mu_0 dI}{2} \frac{R^2 \sin^2\Theta}{(R^2\sin^2\Theta + R^2\cos^2\Theta)^{3/2}} =$$

$$= \frac{\mu_0 \sigma \omega d\Theta}{2} \cdot \frac{R^4 \sin^3\Theta}{R^3} = \frac{\mu_0 \omega \sigma R}{2} \sin^3\Theta \, d\Theta$$

$$\vec{B}_0 = \int_0^\pi dB = \frac{\mu_0 \omega \sigma R}{2} \int_0^\pi \sin^3\Theta \, d\Theta = \frac{2}{3} \mu_0 \vec{\omega} \sigma R$$

3) Найдём линейную плотность токов:

$$i = \frac{dI}{R\,d\Theta} = \sigma \omega R \sin\Theta = i_0 \sin\Theta$$



по т. косинусов:

$$r^2 = R^2 + z^2 - 2Rz\cos\Theta$$

$$dB = \frac{\mu_0 \, dI \cdot R^2 \sin^2\Theta}{2r^3} = \frac{\mu_0 i_0 R^3 \sin^3\Theta \, d\Theta}{2r^3} =$$

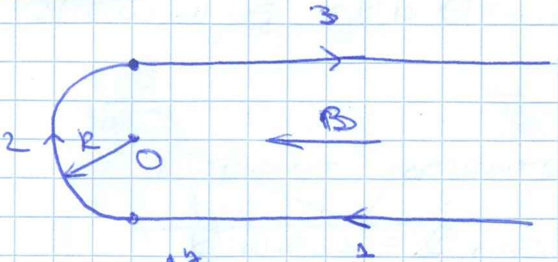
$$= \frac{\mu_0 i_0 R^3 \sin^3\Theta}{2} (R^2 + z^2 - 2Rz\cos\Theta)^{-3/2} d\Theta =$$

$$= \frac{\mu_0 i_0}{2} \frac{\sin^3\Theta \, d\Theta}{(1 + \xi^2 - 2\xi\cos\Theta)^{3/2}}, \quad \xi = \frac{z}{R}$$

$$\vec{B} = \frac{\mu_0 i_0}{2} \int_0^\pi \frac{\sin^3\Theta \, d\Theta}{(1+\xi^2 - 2\xi\cos\Theta)^{3/2}} = \frac{2\mu_0 i_0}{3} =$$

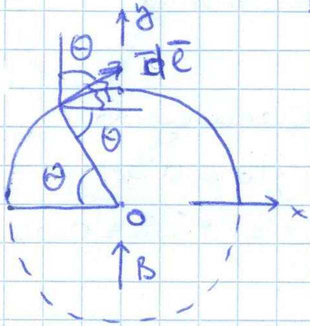
$$= \frac{2}{3} \mu_0 \sigma R \vec{\omega}$$

7.10



$$F_1 = I\,[\vec{l}, \vec{B}] = 0$$

$$F_3 = I\,[-\vec{l}, \vec{B}] = 0$$



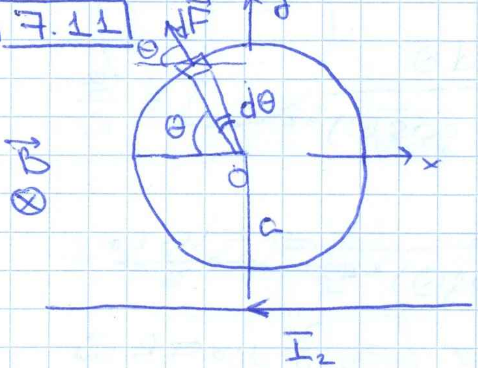
$$\vec{F} = I \oint_L [d\vec{l}, \vec{B}] =$$

$$= I \oint_L \left([dl \cdot \cos\Theta\, \vec{j}, \vec{B}] + [dl \sin\Theta\, \vec{j}, \vec{B}] \right) =$$

$$= I \oint_L \left(R\, d\Theta \cos\Theta \underbrace{[\vec{j}, \vec{B}]}_{=0} + R \sin\Theta\, d\Theta\, [\vec{i}, \vec{B}] \right) =$$

$$= I \int_0^{\pi} BR \sin\Theta\, d\Theta \cdot \vec{i} = 2BRI \cdot \vec{i}$$

7.11



$$B = \frac{\mu_0 I_2}{2\pi (a + R \sin\Theta)}$$

$$dF = I_1 [d\vec{l}, \vec{B}]$$

$$F_y = I_1 \oint_L [d\vec{l} \cdot \sin\Theta, \vec{B}] = I_1 \int_0^{2\pi} B\, R\, d\Theta \cdot \sin\Theta =$$

$$= I_1 \int_0^{2\pi} \frac{\mu_0 I_2}{2\pi} \cdot \frac{R\sin\Theta}{a + R\sin\Theta} d\Theta =$$

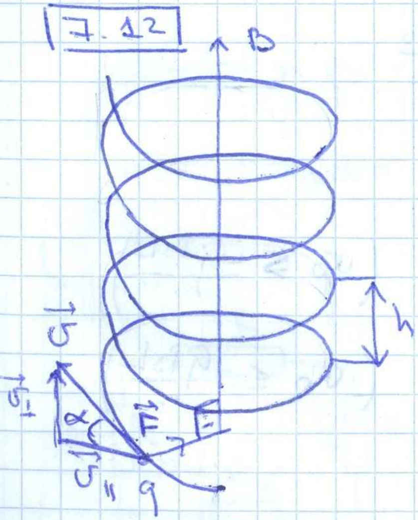
$$= \frac{\mu_0 I_1 I_2}{2\pi} \int_0^{2\pi} \frac{R\sin\Theta}{a + R\sin\Theta} d\Theta =$$

$$= \mu_0 I_1 I_2 \left(-1 + \sqrt{\frac{a^2}{a^2 - R^2}} \right)$$

$$F_x = I_2 \oint_L [\vec{dl}\cos\Theta, \vec{B}] = \frac{\mu_0 I_1 I_2}{2\pi} \int_0^{2\pi} \frac{R\cos\Theta}{a + R\sin\Theta} d\Theta =$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} \ln(a + R\sin\Theta) \Big|_0^{2\pi} = 0$$

$\boxed{7.12}$



Частицы двигаются по спирали с радиусом витка R

$$\begin{cases} \vec{F} = q[\vec{v}, \vec{B}] = qv\sin\alpha B = ma \\ a = \frac{v_\parallel^2}{R} = \frac{v^2\sin^2\alpha}{R} \end{cases}$$

$$\Rightarrow R = \frac{mv\sin\alpha}{Bq}$$

и шагом спирали h:

$$h = v_\perp T = v_\perp \cdot \frac{2\pi R}{v_\parallel} = \frac{2\pi m \cancel{v\sin\alpha}}{Bq\,\cancel{v\sin\alpha}}\, v\cos\alpha =$$

$$= \frac{2\pi m v\cos\alpha}{Bq}$$

Точки максимально удалятся друг от друга на расстояние

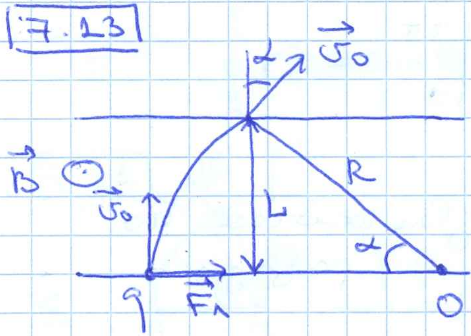
$$d = 2R + 2R = 4R$$

И первый раз встретятся через время

$$T = \frac{2\pi R}{v_\parallel} = \frac{2\pi m\, \cancel{v\sin\alpha}}{Bq\, \cancel{v\sin\alpha}} = \frac{2\pi m}{Bq}$$

на расстоянии h от точки старта

$\boxed{7.13}$



$$\frac{m v_0^2}{R} = q v_0 B \Rightarrow$$

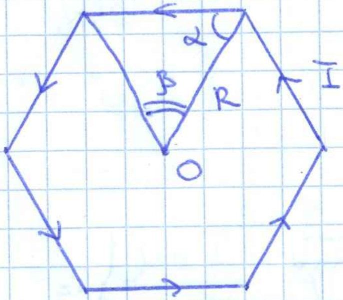
$$\Rightarrow R = \frac{m v_0}{qB}$$

$R > L$: $\sin\alpha = \frac{L}{R} = \frac{LqB}{m v_0}$ $\left(v_0 \geq \frac{qBL}{m}\right)$

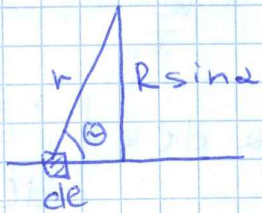
$R \leq L$: $\alpha = \pi$. $\left(v_0 \leq \frac{qBL}{m}\right)$

$\boxed{7.14}$

$$2\alpha = \pi - \beta = \pi - \frac{2\pi}{n} \Rightarrow$$



$$\Rightarrow \quad \alpha = \frac{n-2}{2n}\pi$$



$$dB = \frac{\mu_0 I}{4\pi r^2} d\ell \sin\theta = \left\{ \begin{aligned} & r = \frac{R\sin\alpha}{\sin\theta} \\ & d\ell\sin\theta = r\,d\theta = \frac{R\sin\alpha}{\sin\theta} d\theta \end{aligned} \right\}$$

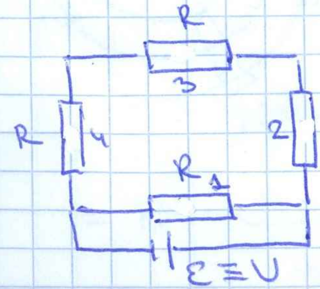
$$= \frac{\mu_0 I \sin\theta}{4\pi R\sin\alpha} d\theta$$

$$B = \frac{\mu_0 I}{4\pi R\sin\alpha}\int_{\alpha}^{\pi-\alpha} \sin\theta\, d\theta = \frac{\mu_0 I}{2\pi R}\operatorname{ctg}\alpha$$

$$B_\Sigma = nB = \frac{\mu_0 n I}{2\pi R}\operatorname{ctg}\alpha, \quad \alpha = \frac{n-2}{2n}\pi$$

$$\operatorname{ctg}\alpha = \operatorname{tg}\left(\frac{\pi}{2} - \alpha\right) = \operatorname{tg}\left(\frac{\pi}{2} - \frac{n-2}{2n}\pi\right) = \operatorname{tg}\frac{\pi}{n}$$

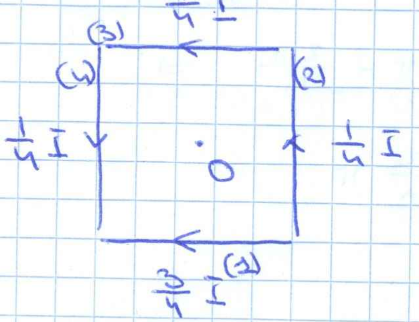
$\boxed{7.15}$

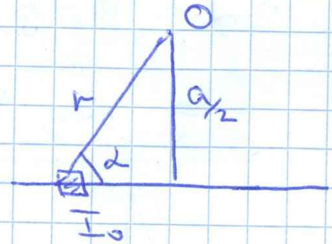


$$U = I_1 \cdot R = I_2 \cdot (R+R+R)$$

пусть через ЭДС течёт ток I:

$$I_1 = \frac{3I}{4} \qquad I_2 = \frac{I}{4}$$





$$dB = \frac{\mu_0 I_0}{4\pi r^2}\sin\alpha\, dl = \left\{ \begin{aligned} & r = \frac{a}{2\sin\alpha} \\ & dl \cdot \sin\alpha = r\, d\alpha \end{aligned} \right\} =$$

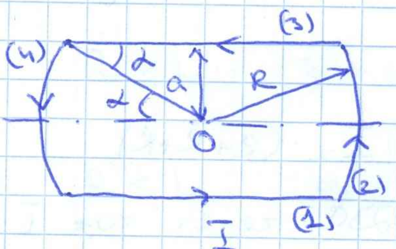
$$= 2\frac{\mu_0 I_0}{4\pi a}\sin\alpha\, d\alpha$$

$$B = 2\frac{\mu_0 I_0}{4\pi a}\int\limits_{\pi/4}^{3\pi/4}\sin\alpha\, d\alpha = \frac{\mu_0 I_0}{\pi a}\cos\frac{\pi}{4}$$

$$B_\Sigma = -B_1 + B_2 + B_3 + B_4 =$$

$$= -\frac{3}{4}\frac{\mu_0 I}{\pi a}\cos\frac{\pi}{4} + 3\cdot\frac{\mu_0 I}{4\pi a}\cos\frac{\pi}{4} = 0.$$

$\boxed{7.16}$

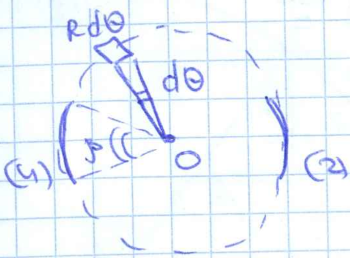


известно, что (7.15)

$$B_1 = B_2 = \frac{\mu_0 I}{2\pi a}\cos\alpha,$$

$$\sin\alpha = \frac{a}{R}$$

$\otimes\, B_\Sigma$



$$\vec{B}_0(O) = \frac{\mu_0 I}{4\pi R^3} \oint_L [d\vec{l}, \vec{R}] =$$

$$= \frac{\mu_0 I}{4\pi R^3} \oint_L dl \cdot R \cdot \vec{k} =$$

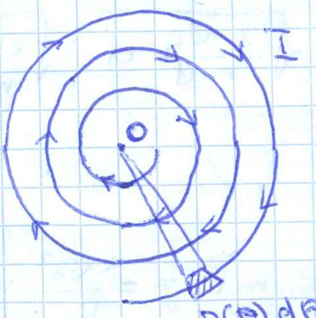
$$= 2\frac{\mu_0 I}{4\pi R^3} \int_0^{\beta} R^2 \, d\varphi \, \vec{k} = \frac{\mu_0 I}{4\pi R} \cdot 2\beta \, ; \; \beta = 2\alpha$$

$$B_\Sigma = B_1 + B_2 + B_0 = 2\frac{\mu_0 I}{2\pi a}\sqrt{1 - \frac{a^2}{R^2}} + \frac{\mu_0 I \alpha}{\pi R} =$$

$$= \frac{\mu_0 I}{R\pi}\left(\frac{\sqrt{R^2 - a^2}}{a} + \arcsin\frac{a}{R} \right)$$

$\boxed{7.17}$

$$\begin{cases} \rho(\theta) = \alpha\theta + \beta \\ \rho(0) = R_2 & \Rightarrow \beta = R_2 \\ \rho(2\pi N) = R_1 & \Rightarrow \alpha = \frac{R_1 - R_2}{2\pi N} \end{cases}$$



$$\vec{B}(O) = \frac{\mu_0 I}{4\pi} \int_L \frac{[d\vec{l}, \vec{\rho}(\theta)]}{\rho^3(\theta)} =$$

$\otimes \, \vec{n}$

$$= \frac{\mu_0 I}{4\pi} \int_0^{2\pi N} \frac{\rho(\theta)\, d\theta \cdot \rho(\theta)}{\rho^3(\theta)} \vec{n} =$$

$$= \frac{\mu_0 I}{4\pi} \vec{n} \int_0^{2\pi N} \frac{d\theta}{\alpha\theta + \beta} = \frac{\mu_0 I}{4\pi\alpha} \vec{n} \ln(\alpha\theta + \beta) \Big|_0^{2\pi N} =$$

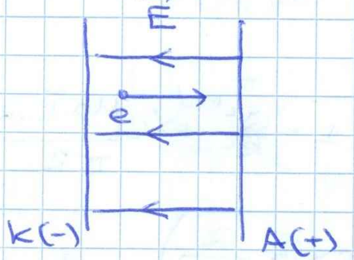
$$= \vec{n} \cdot \frac{\mu_0 I N}{2} \frac{\ln R_1/R_2}{R_1 - R_2}$$

$$\lim_{R_1 \to R_2} B(0) = \lim_{R_1 \to R_2} \frac{\mu_0 I N}{2} \frac{\ln R_1/R_2}{R_1 - R_2} \overset{Л}{=}$$

$$\overset{Л}{=} \frac{\mu_0 I N}{2} \lim_{R_1 \to R_2} \frac{\frac{R_2}{R_1} \cdot \frac{1}{R_2}}{1} = \frac{\mu_0 I N}{2 R_1}$$

$\boxed{7.18}$ Разобьём задачу на 3 этапа:

1) Электрическое поле



$$\begin{cases} U = Ed \\ F = Eq = ma \end{cases} \qquad |e| \equiv q$$

$$\Rightarrow a = \frac{Eq}{m} = \frac{Uq}{md}$$

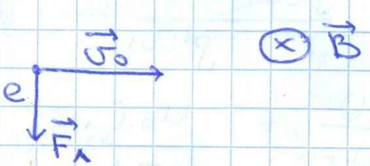
$$v_0 = at$$

$$d = \frac{at^2}{2} \Rightarrow t^2 = \frac{2d}{a} = \frac{2md^2}{Uq}$$

$$\Rightarrow t = d\sqrt{\frac{2m}{Uq}} = \sqrt{\frac{2d}{a}}$$

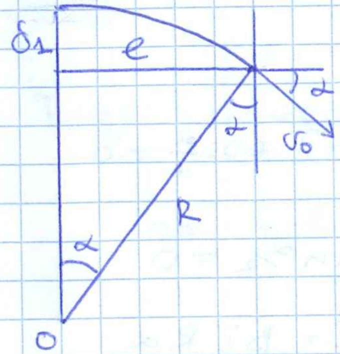
$$v_0 = at = a\sqrt{\frac{2d}{a}} = \sqrt{2da} = \sqrt{\frac{2Uq}{md} d} = \sqrt{\frac{2Uq}{m}}$$

2) Магнитное поле



$$F_л = Bqv_0 = ma = \frac{mv_0^2}{R} \Rightarrow$$

$$\Rightarrow R = \left(\frac{Bq}{mv_0}\right)^{-1} = \frac{mv_0}{Bq}$$



$$\delta_1 = R - R\cos\alpha;$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{e^2}{R^2}}$$

$$\delta_1 = R\left(1-\sqrt{1-\frac{e^2}{R^2}}\right) =$$

$$= \frac{m\upsilon_0}{Bq}\left(1-\sqrt{1-\frac{B^2q^2e^2}{m^2\upsilon_0^2}}\right)$$

$$\upsilon_1 = \upsilon_0\cos\alpha \qquad \upsilon_2 = \upsilon_0\sin\alpha$$

3) Свободный полёт:

$$D = \upsilon_0\cos\alpha\, t \Rightarrow t = \frac{D}{\upsilon_0\cos\alpha}$$

$$\delta_2 = \upsilon_0\sin\alpha\, t = D\,\mathrm{tg}\,\alpha = D\cdot\frac{e}{\sqrt{R^2-e^2}}$$

$$\delta = \delta_1+\delta_2 = \frac{m}{Bq}\cdot\sqrt{\frac{2Uq}{m}}\left(1-\sqrt{1-\frac{B^2q^2e^2}{m^2\upsilon_0^2}}\right) + D\frac{e}{\sqrt{R^2-e^2}}$$

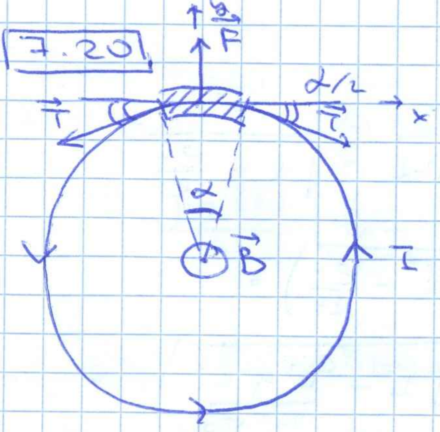
$$= \frac{1}{B}\sqrt{\frac{2Um}{q}}\left(1-\sqrt{1-\frac{B^2q^2e^2}{m^2\upsilon_0^2}}\right) +$$

$$+ D\frac{1}{\sqrt{\frac{m^2\upsilon_0^2}{B^2q^2e^2}-1}} =$$

$$= \frac{1}{B}\sqrt{\frac{2Um}{q}}\left(1-\sqrt{1-\frac{B^2q^2e^2}{m^2}\cdot\frac{m}{2Uq}}\right) +$$

$$+ D\frac{1}{\sqrt{\frac{m^2}{B^2q^2e^2}\cdot\frac{2Uq}{m}-1}}$$

$\boxed{7.20}$



$\alpha \ll 1$

$x:\quad T\cos\frac{\alpha}{2} - T\cos\frac{\alpha}{2} = 0$

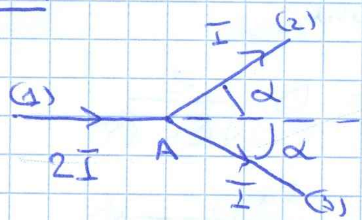
$y:\quad F - 2T\sin\frac{\alpha}{2} = 0$

$$F = BI\,dl = BIR\alpha$$

$$BIR\alpha = 2T\sin\frac{\alpha}{2} \approx T\alpha$$

$$\Rightarrow \quad I = \frac{T}{BR}$$

$\boxed{7.8}$



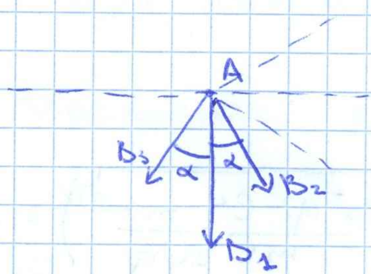
из 7.14 известно, что:

$$B_{(1)} = 2B_{(2)} = 2B_{(3)} =$$

$$= \frac{\mu_0 2I}{4\pi a}\cdot\int_0^{\pi/2}\sin\theta\,d\theta =$$

$$= \frac{\mu_0 2I}{4\pi a}.$$

Легко видеть, что:



$$B_\Sigma = B_1 + 2B_2\cos\alpha =$$

$$= \frac{\mu_0 I}{2\pi a}(1+\cos\alpha)$$

7.5 Из зад. 7.4 известно, что магн. инд. внутри провода равна 0.

Тогда исходная система равносильна такой системе.

по поверхности провода течёт ток $\underbrace{}_{}$ с учётом тонкой щели

$$\hat{I} = I + I_d$$

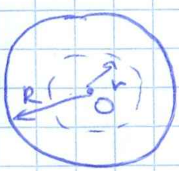
по поверхности щели течёт ток $-I_d$.

$$I_d = \frac{I}{2\pi R} \cdot d = i d$$

Тогда магнитная индукция будет создаваться только щелью. (проводом)

$$B = \frac{\mu_0 I_d}{2\pi r} = \frac{\mu_0 I}{4\pi^2 R r} d$$

7.4 Воспользуемся т. о циркуляции магнитного поля.



$\odot I$ 1) $r < R$:

$$B \cdot 2\pi r = 0 \Rightarrow B = 0$$

2) $r \geq R$:

$$B \cdot 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}.$$

7.8 По т. о циркуляции магн. поля:

1) $r < R$: $B \cdot 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$

2) $R \leq r$: $B \cdot 2\pi r = 0 \Rightarrow B = 0$.